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A NOTE ON THE CORE OF THE OVERLAPPING GENERATIONS MODEL

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In this note it is shown that, without certain restrictions on the coalitions that may form, the core of the overlapping generations model may be empty. The introduction of money, while expanding the trading possibilities, does not eliminate the problem.

1. The first theorem of welfare economics states that Walrasian equilibria are Pareto optimal. The overlapping generations model introduced by Samuelson (1958) provides an example in which this theorem fails. Shell (1975) has argued persuasively that the essential condition responsible for this failure is the 'double infinity' of consumers and commodities.

In this letter, we examine a second proposition in general equilibrium theory. We show that the set of Walrasian equilibria in an overlapping generations model may not belong to the core. In fact, we prove that the core may be empty.

The basic reason for this result appears to be the 'double infinity' rather than the limited opportunities for intertemporal exchange. The introduction of paper assets (money), while expanding the trading possibilities, does not eliminate the problem. The core may remain empty.

2. Assume a countable infinity of agents, one born in each time period t. Each agent a_t lives for two periods, t and $t+1$. In each period only one good is consumed, indexed by that period. All agents are identical. Let w_y be agent a_t's endowment of good t and w_0 be agent a_{t-1}'s endowment of

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good t. Let $(c_y(t),c_0(t+1))$ be agent a_t's consumption vector yielding utility $U(c_y(t),c_0(t+1))$.

In order to give this economy a beginning, we denote by a_0 the old person living in the first period. He has an endowment w_0 of good 1 and a utility function $U^0(c(1))$.

Theorem 1. If, for all $a_t, t>0$,

$$(U_y,U_0)=\frac{\partial U}{\partial C_y}(w_y,w_0)\Big/\frac{\partial U}{\partial C_0}(w_y,w_0)<1$$

the core is empty.

Proof. Let W be the endowment allocation. It cannot belong to the core since the coalition of all agents can reallocate their endowments among themselves and make everyone better off. That is, there exists a feasible sequence of transfers $\{e_t\}_{t=1}^{\infty}$ such that

$$U(w_0+e_1)>U(w_0), \tag{1}$$

$$U(w_y-e_t,w_0+e_{t+1})>U(w_y,w_0), \quad t=1,2,3\ldots \tag{2}$$

The existence of such a sequence is easily demonstrated by expanding (2) about the endowment point. Discarding higher order terms, we find

$$e_{t+1}>(U_y/U_0)e_t>(U_y/U_0)^t e_1. \tag{3}$$

Since $(U_y/U_0)<1$, there exists a sequence $\{e_t\}_{t=1}^{\infty}$ satisfying (1) and (2) which is bounded. Proper choice of e_1 ensures feasibility.

If $(c_y(t),c_0(t+1))$ is a feasible allocation not equal to the endowment allocation W, then there exists an agent $a_{t'}$ who either (i) gives a portion of his endowment of good t' to agent $a_{t'-1}$ or (ii) gives a portion of his endowment of good $t'+1$ to agent $a_{t'+1}$. In case (i) the allocation cannot belong to the core since the coalition $\{a_t\}_{t=t'}^{\infty}$ can provide an allocation in which agent $a_{t'}$ gets $c_y(t')=w_y$, $c_0(t')=c_0(t')$, making him better off, and in which the consumption of agents a_t, $t=t'+1,\ldots$ remains unchanged. A similar construction for case (ii) demonstrates that $(c_y(t),c_0(t+1))$ is not in the core.

Q.E.D.

3. Money, or some other form of paper asset, is a mechanism by which an economy can achieve a Pareto efficient Walrasian equilibrium. However, the introduction of money into the model implies a special kind of social contract. All agents agree that only a_0, the first generation, can create paper assets. Agent a_0 has the only valid claim to the extra utility the economy can create by transferring endowment 'backwards' from the infinite future. Thus, the overlapping generations model with money implicitly restricts the types of coalition structures that can be generated.

It is instructive to examine precisely the effect upon the core of various restrictions on coalition structures. The emptiness of the core is basically due to the existence of infinite member coalitions. If we restrict the coalition structure by allowing only finite member coalitions, the core consists of the endowment allocation. This can be seen by noting that in a finite set of agents, an agent can have his utility increased only by transfers from future or past generations and that such transfers must leave either the 'first' or the 'last' member of the coalition with less utility than his endowment.

Let us now suppose that there is an agent with an infinite life and that he can make agreements with any other agent. Further, let us suppose that the infinite lived agent has no endowment and is satiated at zero. Then the Pareto optimal allocation is in the core in the following sense: the infinite lived agent can make a series of contracts with finite lived agents which has the effect of reallocating the endowment optimally, and no agent has incentive to break out of his contract. However, if infinite member coalitions are permitted, then each finite lived agent will try to block this allocation by forming a coalition in which he makes no contribution to anyone but receives a transfer.

References

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